

# 3 Theoretical Basis for SAM.sed Calculations

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## Purpose

Sediment transport functions can be used to calculate the bed material portion of the sediment discharge rating curve. This rating curve can then be used in the Flow-Duration Sediment-Discharge Rating Curve Method, programmed in SAM.yld, to calculate the bed-material sediment yield. Various sediment transport equations have been programmed into SAM.sed.

EM 1110-2-4000, "Sediment Investigations of Rivers and Reservoirs," provides a good description of the mechanics of sediment transport (USACE 1989).

## General

SAM.sed will calculate a sediment discharge rating curve based on hydraulic conditions and the bed gradation. The hydraulic input data for SAM.sed can be calculated using SAM.hyd or it can be determined external to the program and input directly (SAM.m95 can be used only in DOS mode). SAM.sed will create a partial input data set for SAM.yld. There is also a module, SAM.aid, which provides guidance on the selection of transport functions.

Sediment transport functions in SAM must be used with care. There is no allowance for variability in the size class distribution over time or space. In natural rivers the size class distribution of bed material varies with discharge, reach, time of year, and other temporal factors. SAM's use of a fixed, average size class distribution for all calculations presents the possibility that the calculated transport rates are not truly representative of the natural river. The procedure in HEC-6, which integrates processes over several cross sections and provides a continuity equation for sediment movement, will consequently produce a more reliable result. SAM provides reasonable time-averaged results in cases where the river is in general equilibrium; that is, it is neither aggrading or degrading. It is very important that the user correctly prescribe a representative bed material gradation.

## Sediment Transport Functions

The following sediment transport functions have been incorporated into SAM. Except for Brownlie, these functions are also available in HEC-6. Some are identified as "d<sub>50</sub>" which means the original version was for a single grain size. That capability is provided in SAM.sed. The HEC-6 versions of the functions calculate by grain size class and that capability is also in SAM.sed.

Table 7. Sediment Transport Functions available in SAM.sed.

ACKERS-WHITE.	MPM (1948), D50
ACKERS-WHITE, D50	PARKER
BROWNLIE, D50	PROFITT (SUTHERLAND)
COLBY	SCHOKLITSCH
EINSTEIN (BED-LOAD)	TOFFALETI.
EINSTEIN (TOTAL-LOAD)	TOFFALETI-MPM
ENGELUND-HANSEN	TOFFALETI-SCHOKLITSCH
LAURSEN (COPELAND)	YANG
LAURSEN (MADDEN), 1985	YANG, D50
MEYER-PETER and MULLER (MPM) (1948)	VAN RIJN

Ackers-White is a version of Ackers-White (1973) which has been modified, at WES, for multiple grain size calculations on sand and/or gravel bed streams. Modifications made by Ackers (1993) have been incorporated.

Ackers-White, D50 is a single grain size function.

Brownlie (1981) is a single grain size function for sand transport.

Colby (1964) is a version of Colby's single grain size function which has been modified at WES for multiple grain size calculations. It is valid for sand transport in streams and small rivers.

Einstein (Bed-load) (1950) is a multiple grain size function used to calculate the bed-load discharge of sand and/or gravel bed streams. The hiding factor has been modified at WES, incorporating the work of Pemberton (1972) and Shen and Lu (1983).

Einstein (Total-load) (1950) is a function that extends the Einstein bed-load calculations to include suspended load by grain size classes and sums them to get the total load.

Engelund (Hansen) (1967) is a version of Engelund-Hansen D<sub>50</sub> which has been modified at WES for multiple grain size calculations on sand bed streams.

Laursen (Copeland) (Copeland and Thomas, 1989) is a modification to Laursen's (1958) multiple grain size function, extending its range to larger gravel sizes.

Laursen (Madden) (Madden, 1993) is a multiple grain size function modified by Madden for sand bed transport. It has been used for mixtures of sand and gravel. There is a 1963 modification in HEC-6 that is not available in SAM.

MPM – Meyer-Peter and Muller (1948) is a multiple grain size function for gravel bed rivers. It is not valid when appreciable suspended load is present.

MPM, D50 -- Meyer-Peter and Muller (1948), D50 is a version of the multiple grain size function MPM which has been modified, at WES, using a single grain size function.

Parker (1990) is a version of Parker's multiple grain size function. It can be applied to poorly sorted gravel bed streams. Finer sizes, less than 2 mm, must be excluded from the specified surface size distribution and the gradation must be 100% defined, i.e., there must be a size for which 0% of the material is finer. The bed material sizes used must be representative of the coarse upper layer of the bed.

Profitt (Sutherland) (1983) is a multiple grain size function modification of the Ackers-White formula. It can be used on sand and/or gravel bed streams.

Schoklitsch (1930) is a version of the Schoklitsch single grain size function that has been modified at WES for multiple grain size calculations. It is applicable to sand and gravel bed streams which do not have considerable amounts of suspended sediment transport.

Toffaletti (1968) is a multiple grain size function for sand bed rivers. It is not valid for gravel transport.

Toffaletti-MPM is a combined function for sand and gravel bed streams. Sediment transport is calculated using both functions by size class. Calculated bed load from the Toffaletti function is compared to the total calculated by MPM and the larger is used for bed load. Suspended load is then calculated using Toffaletti.

Toffaletti-Schoklitsch is a combined function for sand and gravel bed streams. Sediment transport is calculated using both functions by size class. Calculated bed load from the Toffaletti function is compared to the total calculated by Schoklitsch and the larger is used for bed load. Suspended load is then calculated using Toffaletti.

- Van Rijn is based on Van Rijn (1984 a, b). Some of the modifications made by Spasojevic and Holly (1994) for CH3D-SED are incorporated. The most significant change is the multiple-grain grain size treatment. A hiding factor is applied to the bed load transport, but not the suspended load transport. Shield's parameter is calculated using Van Rijn's equations. The suspended sediment concentration profile is calculated using equations proposed in Van Rijn's paper (1984b). Recommended use is for grain sizes between 0.1 and 0.5 mm.
- Yang (1973, 1984) is a version of Yang, D50, which has been modified, at WES, for multiple grain size calculations in sand and gravel bed streams (less than 10 mm).
- Yang, D50 is a single grain size function for sand and gravel transport in streams and small rivers.

This is not an exhaustive list of transport functions. These were selected based on both corporate experience and recommendations from the literature. There is no implication that those not selected are deficient. The objective in SAM.sed was to provide designers with a "few" acceptable methods whose use could be supported. The criteria for selection were

- a. to cover a broad range of particle sizes;
- b. to cover a broad range of hydraulic conditions;
- c. to calculate sediment transport by partitioning the mixture into size classes and summing the rate of each to get the total, except when  $d_{50}$  functions are requested; and
- d. to have a history of being reliable when used within the range of data for which each was calibrated.

The Brownlie function is included here because it is used in the analytical method for calculating channel width, depth and slope in SAM.hyd. It is a single grain size function.

## Calculations

The functional form for sediment transport equations is

$$GS_i = \ddot{u}(V, D, S_e, B_e, d_e, \rho_s, G_{sf}, d_s, i_b, \rho_f, T) \quad \text{Equation 3-1}$$

where

$GS_i$  = transport rate for size class  $i$   
 $V$  = average flow velocity  
 $D$  = effective depth of flow  
 $S_e$  = energy slope  
 $B_e$  = effective width of flow (width of portion of cross section which is transporting bed material sediment)  
 $d_e$  = effective particle size for the mixture  
 $\rho_s$  = density of sediment particles  
 $G_{sf}$  = grain shape factor  
 $d_s$  = geometric mean of particles in size class  $i$   
 $i_b$  = fraction of size class  $i$  in bed  
 $\rho_f$  = density of fluid  
 $T$  = temperature of fluid

Of particular interest are the groupings of terms: hydraulic parameters ( $V$ ,  $D$ ,  $S_e$ ,  $B_e$ ), sediment particle parameters ( $d_e$ ,  $\rho_s$ ,  $G_{sf}$ ), sediment mixture parameters ( $d_s$ ,  $i_b$ ) and fluid properties ( $\rho_f$ ,  $T$ ).

Not all listed parameters are used in all sediment transport functions. In SAM.sed, the specific gravity of sediment defaults to 2.65, and the particle shape factor defaults to 0.667. However, these defaults can be over-ridden.

## Procedure for Calculating Sediment-Discharge Rating Curve

The steps in calculating a sediment-discharge rating curve from the bed-material gradation are:

1. assemble field data (cross sections and bed gradations)
2. develop representative values for roughness coefficients, geometry, and bed gradation from the field measurements
3. calculate the stage-discharge rating curve accounting for possible regime shifts due to bed-form change
4. calculate the bed-material sediment-discharge rating curve using hydraulic parameters from the stage-discharge calculation.

SAM.sed can be used for step 4. The input information required by SAM.sed, from step 3, can be obtained from SAM.hyd or HEC-2.

## Data Ranges used in Development of Sediment Transport Functions

The range of data used in the development of the sediment transport functions that have been included in SAM.sed are summarized in Tables 9 through 21. These summaries are based on the authors' stated ranges when presented in their original papers. Otherwise, the summaries were determined based on the author's description of his data base in combination with the data listings of Brownlie (1981) or Toffaleti (1968).

Table 9. Ackers-White Transport Function

Parameter	Flume Data
Particle Size Range, mm	0.04 - 7.0
Specific Gravity	1.0 - 2.7
Multiple Size Classes	no
Velocity, fps	0.07 - 7.1
Depth, ft	0.01 - 1.4
Slope, ft per ft	0.00006 - 0.037
Width, ft	0.23 - 4
Water Temperature, Deg F	46 - 89

Table 10. Brownlie Transport Function

Parameter	River Data	Flume Data
Particle Size Range, mm	0.086 - 1.4	0.088 - 1.4
Multiple Size Classes	no	no
Velocity, fps [calculated]	1.2 - 7.9	0.7 - 6.6
Depth, ft	0.35 - 57	0.11 - 1.9
Slope, ft per ft	0.00001 - 0.0018	0.00027 - 0.017
Width, ft	6.6 - 3640	0.83 - 8.0
Water Temperature, Deg F	32 - 95	35 - 102

Table 11. Colby Transport Function

Parameter	Data Range
Particle Size Range, mm	0.18 - 0.70
Multiple Size Classes	no
Velocity, fps	0.70 - 8.0
Depth, ft	0.20 - 57
Slope, ft per ft	0.000031 - 0.010
Width, ft	0.88 - 3000
Water Temperature, Deg F	32 - 89
Correction for Fines, ppm	yes

Table 12. Einstein Transport Function

Parameter	Flume Data
Particle Size Range, mm	0.78 - 29
Multiple Size Classes	yes
Velocity, fps	0.9 - 9.4
Depth, ft	0.03 - 3.6
Slope, ft per ft	0.00037 - 0.018
Width, ft	0.66 - 6.6
Water Temperature, Deg F	not reported

Table 13. Engelund-Hansen Transport Function

Parameter	River Data	Flume Data
Particle Size Range, mm		
Multiple Size Classes		
Velocity, fps		
Depth, ft		<b>DATA NOT AVAILABLE YET</b>
Slope, ft per ft		
Width, ft		
Water Temperature, Deg F		

Table 14. Laursen(Copeland) Transport Function

Parameter	River Data	Flume Data
Median Particle Size Range, mm	0.08 - 0.70	0.011 - 29
Multiple Size Classes	yes	yes
Velocity, fps	0.068 - 7.8	0.70 - 9.4
Depth, ft	0.67 - 54	0.03 - 3.6
Slope, ft per ft	0.0000021 - 0.0018	0.00025 - 0.025
Width, ft	63 - 3640	0.25 - 6.6
Water Temperature, Deg F	32 - 93	46 - 83

Table 15. Laursen(Madden),(1985)

Parameter	Data Range
Particle Size Range, mm	0.04 - 4.8
Multiple Size Classes	yes
Velocity, fps	0.85 - 7.7
Depth, ft	0.25 - 54
Slope, ft per ft	0.00001 - 0.1
Width, ft	3 - 3640
Water Temperature, Deg F	36 - 90

Table 16. Meyer-Peter and Muller, 1948, Transport Function

Parameter	Data Range
Particle Size Range, mm	0.4 - 29
Particle Specific gravity	1.25 - 4
Multiple Size Classes	yes
Velocity, fps	1.2 - 9.4
Depth, ft	0.03 - 3.9
Slope, ft per ft	0.0004 - 0.02
Width, ft	0.5 - 6.6
Water Temperature, Deg F	not published

Table 17. Parker Transport Function

Parameter	River Data	Flume Data
Median Particle Size Range, mm	18 - 28	
Total Particle Size Range, mm	2 - 102	
Multiple Size Classes	yes	
Velocity, fps	2.6 - 3.7	
Depth, ft	1.0 - 1.5	
Slope, ft per ft	0.0097 - 0.011	
Width, ft	16 - 20	
Water Temperature, Deg F	41 - 44	

Table 18. Profitt(Sutherland) Transport Function

Parameter	River Data
Particle Size Range, mm	2.90 - 12
Multiple Size Classes	yes
Velocity, fps	2.00 - 3.4
Depth, ft	0.35 - 0.84
Slope, ft per ft	0.003
Width, ft	2.00
Water Temperature, Deg F	59 - 63

Table 19. Schoklitsch Transport Function

Parameter	Data Range
Particle Size Range, mm	0.3 - 4.9
Multiple Size Classes	no
Velocity, fps	0.8 - 4.5
Depth, ft	0.037 - 0.74
Slope, ft per ft	0.00012 - 0.055
Width, ft	0.23 - 2.0
Water Temperature, Deg F	not published

Table 20. Toffaleti Transport Function

Parameter	River Data	Flume Data
Median Particle Size Range, mm	0.095 - 0.76	0.91 - 0.45
Total Particle Size Range, mm	0.062 - 4	0.062 - 4
Multiple Size Classes	yes	yes
Velocity, fps	0.7 - 7.8	0.7 - 6.3
Hydraulic Radius, ft	0.7 - 56.7	0.07 - 1.1
Slope, ft per ft	0.000002 - 0.0011	0.00014 - 0.019
Width, ft	63 - 3640	0.8 - 8
Water Temperature, Deg F	32 - 93	40 - 93

Table 21. Van Rijn Transport Function

Parameter	Sand Data	Gravel Data
Particle Size Range, mm		
Multiple Size Classes		
Velocity, fps		
Depth, ft	<b>DATA NOT AVAILABLE YET</b>	
Slope, ft per ft		
Width, ft		
Water Temperature, Deg F		

Table 22. Yang Transport Function

Parameter	Sand Data	Gravel Data
Particle Size Range, mm	0.15 - 1.7	2.5 - 7.0
Multiple Size Classes	no	no
Velocity, fps	0.8 - 6.4	1.4 - 5.1
Depth, ft	0.04 - 50	0.08 - 0.72
Slope, ft per ft	0.000043 - 0.028	0.0012 - 0.029
Width, ft	0.44 - 1750	0.7 - 1.3
Water Temperature, Deg F	32 - 94	not reported

## Correction for Sand Transport in High Concentration of Fines

Colby (1964) showed a significant increase in the transport capacity of sands when high concentrations ( $C_f > 10,000$  ppm) of fine sediments (wash load) were present. The correction factor ranged up to 100 times the normal transport as the water depth and fines-concentration increased. The range of parameters tested is given in the following table.

Parameter	Range of data
concentrations	up to 200,000 ppm
velocities	up to 10 fps
depths	up to 100 ft
grain sizes	up to 0.9 mm.

At present, only the Colby function contains a correction for fines in SAM. In HEC-6, the Colby correction factor is applied to all sediment transport equations except Toffaleti, when Toffaleti is not combined with other equations.

## Importance of Bed-material Gradation Designation

In the calculation of sediment transport, the designated bed gradation controls the calculated sediment discharge. The rate of transport increases exponentially as the grain size decreases, as shown in Figure 3.1. Therefore, bed-material gradations must be determined carefully. Techniques for selecting a representative sample are discussed in EM 1110-2-4000 (USACE 1989). Due to the sensitivity of transport calculations to the grain size, especially the finer sizes, Einstein (1950) recommended excluding the finest 10 percent of the sampled bed gradation from calculation of the total bed-material load.

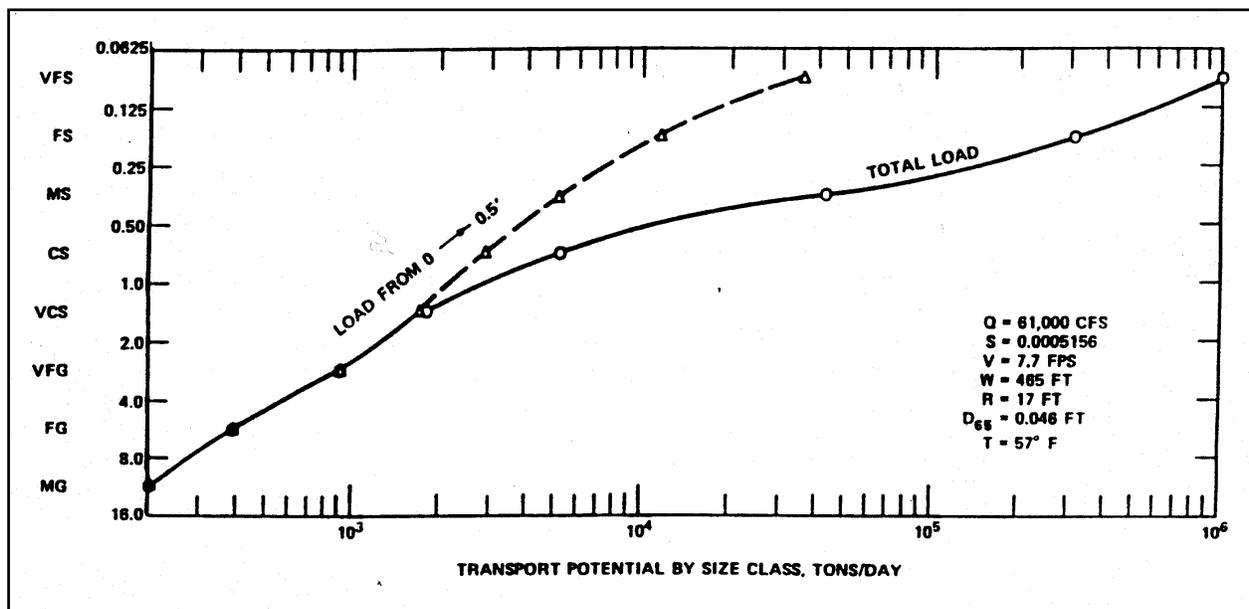


Figure 3-1. Variation of sediment transport with grain size (from EM 1110-2-4000 (USACE 1989), Figure 10-7)